A generalized twistor dynamics of the D=3 SUSY systems.

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A generalization of the twistor shift procedure to the case of superparticle interacting with the background D=3 N=1 Maxwell and D=3 N=1 supergravity supermultiplet is considered. We investigate twistor shift effects and discuss the structure of the resulting constraint algebra.

1 Introduction

The hope to solve the problem of the covariant quantization of Green – Schwarz superstrings is connected now with a success and further development of the twistor – like and relative approaches [1] to the theory of extended supersymmetric objects. In this respect of great importance is the studying of twistor – like formulations of superparticles which are a limiting case of superstrings theory.

Two attractive superparticle formulations in the framework of the twistor – like approach intensively investigated nowadays are the Ferber – Shirafuji [5, 6] and the Sorokin – Tkach – Volkov – Zheltukhin (STVZ) [2] formulation.

The STVZ action is a version of a massless relativistic superparticle with intristic incorporation of twistor – like commuting spinors into the theory.

The presence of the twistor – like spinors gave the possibility of decomposing the set of constraints of the theory onto the first and second class without

introducting the additional "Stuckelberg" variables [14, 15] by projecting it onto the twistor directions [2] and solved the problem of the infinite reducibility of the κ – symmetry by replacing the former with the superconformal worldline supersymmetry [3] being irreducible by definition.

Several versions of twistor – like supersymmetric particles and heterotic strings based on the STVZ action have been constructed in D=3,4 and 6 space – time dimensions [2, 19, 20, 12, 21]. However, the generalization of the STVZ action to the case of a D=10 superparticle posseses another infinite reducible symmetry [12] that can be an obstacle for applying the BRST – BFV – BV quantization scheme.

Perhaps, the solution to this problem can be found by considering the Ferber [5] and Shirafuji [6] twistor superparticle action classically equivalent to the STVZ action. Unfortunately, a worldline supersymmetric version of the Ferber – Shirafuji action [3] is part of a more general action describing the so called spinning superparticle [13] and does not describe the usual N=1 Brink – Schwarz superparticle, but a model with not well defined physical content because the target space is not the conventional superspace, but one with additional θ – translation. However, we may consider a sector of this theory corresponding to the N=1 Brink – Schwarz superparticle [9] and containing a particular solution to the equations of motion analogous to that considered in [3]. The problem of the appropriate worldline supersymmetrization of the Ferber – Schirafuji action has been solved recently in [10].

In both, the Ferber – Shirafuji and STVZ action, twistor – like variables are non-propagating auxiliary degrees of freedom. But we can "animate" their by adding new terms depending on twistor – like variables and their proper – time derivatives. The terms of such kind can arise as a result of particle interaction with quantum fields. Since twistor – like variables are commuting spinors it seems that the introduction of new terms breaks the Pauli spin – statistics theorem. But in the paper [8], where group – theoretical description of semion dynamics in D=3 was considered, it was shown that an "animation"

of twistor – like variables in the case of the STVZ action describing a massive particle leads to arising states with spin 1/4 and 3/4. Thus, the introduction of the new term can drastically change the situation and lead to models for describing more general quantum objects.

On the other hand the twistor dynamics described by the Ferber – Shirafuji action can be modified by analogous terms [7]. In the case of a free particle due to a so called twistor shift procedure a generalized dynamics is equivalent to the standard one, but in the case of particle interaction with background fields it leads to a modification of the interaction which becomes nonminimal and is characterized by infinite series of terms. The modified interaction contains a field strength tensor and its higher derivatives and becomes nonlocal. In the case of a D=3 Maxwell field background the first term in this series describes a particle possessing an anomalous magnetic moment. As result, one arrives at a model relevant to Chern – Simons systems and self – interacting anyons (see,for example, references in [8]).

Such fundamental notions as locality and causality are basically connected with the concept of background fields. There are two principal kinds of locality and causality, namely the worldline (worldsheet) and target space ones. The string ideology is based on the worldsheet locality while field theory deals with locality in target space. Since all physical fields in the framework of string concept are nothing but superstring oscillations it is very important to understand the connection between worldsheet and target space locality. String interaction generate an effective interaction of fields which is characterized by terms with higher derivatives of the field potentials, thus leading to non-local interactions. The presence of these new terms affects the structure of field equations of motion, integrability conditions etc. An anologous situation is reproduced in the generalized twistor dynamics.

From this point of view the main motivation of our research is to study the twistor shift procedure in the case of a superparticle interacting with background superfields, its effects and structure of the constraint algebra.

The article is organized as follows. Section 2 briefly describes twistor dynamics and its generalization. In section 3 we investigate twistor shift effects in the system describing superparticle interacting with D=3 background Maxwell supermultiplet and discuss the constraint algebra. In section 4 we consider as more complicated case superparticle in D=3 supergravity background.The discussion of the obtained results is given in the section Conclusion.

2 Twistor dynamics and its generalization.

Twistor – like formulation of D=3 particle is based on the following action for D=3 massless spinless relativistic particle proposed by Sorokin, Tkach, Volkov and Zheltukhin [2]

$$S_{S.T.V.Z} = \int dt\, p_m(\dot{x}^m - \lambda\gamma^m\lambda) \tag{1}$$

This action reproduces the Cartan – Penrose momentum representation [1]

$$p_m = \lambda\gamma_m\lambda \tag{2}$$

as constraint solution

$$p^2 = 0 \tag{3}$$

on the equation of motion.

With the help of the representation (2) it is easy to prove classical equivalence of the action (1) to the twistor particle action proposed by Ferber and Shirafuji [5, 6]:

$$S_{twistor} = \int dt\, \lambda\gamma_m\lambda\dot{x}^m \tag{4}$$

where as well as in the action (1) λ is the commuting Majorana spinors.

The further generalization of the massless relativistic particle twistor dynamics is connected with ref. [7] where with the aim of "animation" of the twistor variables being non-propagating auxiliary degrees of freedom the action (4) was modified by addition of a term depending on the twistor variables and

their proper – time derivatives the simplest variant of which has the following form:

$$S_{additional} = \int d\tau\, l \lambda_\alpha \dot{\lambda}_\beta \varepsilon^{\alpha\beta} \tag{5}$$

The Hamilton analysis of the system described by the action

$$S_{general} = S_{twistor} + S_{additional} \tag{6}$$

shows that due to the existence of the nontrivial transformation to the new space – time coordinates,

$$\hat{x}^{\alpha\beta} = x^{\alpha\beta} + \frac{l}{2(\lambda\mu)}(\lambda^\alpha \mu^\beta + \lambda^\beta \mu^\alpha) \tag{7}$$
$$x^{\alpha\beta} \equiv x^m (\gamma_m)^{\alpha\beta}$$

containing parameter l with the dimension of length, a generalized twistor dynamics in the free case is equivalent to the standard one, i.e.

$$S_{general} = \int d\tau\, \lambda_\alpha \lambda_\beta \dot{\hat{x}}^{\alpha\beta} \tag{8}$$

The transformation (7) is usually called the twistor shift transformation.

The situation drastically changes in the case of particle interaction with an external background field since the field potentials depending on the space – time coordinates at the transition to the new space – time coordinates $\hat{x}$ reduce to the arising of the infinite power series of the nonlocality parameter with dimension of length, containing field strength tensor and its higher derivatives. Thus, in the framework of the generalized dynamics the interaction changes from minimal scheme to nonminimal.

In the case of particle interaction with the Maxwell background field the interaction modification in the first power of l has the following form

$$\mathcal{A}_m \to \mathcal{A}_m + l\varepsilon_{mnk} F^{nk} \tag{9}$$

where F^{nk} is the Maxwell strength tensor, ε_{mnk} is the Levi – Chivita tensor, and is related to the interaction of particle having an anomalous magnetic moment with Maxwell field. The theories of such kind are intensively investigated nowadays in connection with possible applications to the theory of hight temperature superconductivity and fractional quantum Hall effect.

3 Supertwistor shift in the presence of the background Maxwell field.

A generalized dynamics of superparticle interacting with the Maxwell background superfield is described by the following action

$$S = \int d\tau\, d\eta\, (iE^{-1} DE^{\alpha} DE^{\beta} (\gamma_m)_{\alpha\beta} DE^m + lE^{-1} \dot{E}^{\alpha} DE_{\alpha} + DE^A \mathcal{A}_{\mathcal{A}}) \qquad (10)$$

where $D = \partial_\eta + i\eta\partial_\tau$ is the covariant derivative of the "small" world – line SUSY parametrized by proper – time τ and its grassman superpartner η

$$DE^A = (Dz^M) E_M^A$$

$E_M^A(z)$ is the standard superdreinbein of the flat superspace with coordinates $z^M = (X^m, \Theta^\alpha)$

$$X^m = x^m + i\eta\chi^m; \Theta^\alpha = \theta^\alpha + \eta\lambda^\alpha \qquad (11)$$

being a scalar under "small" SUSY transformations.

$E^{-1} = e^{-1} - i\eta\hat{\psi}/e^2$ is the analog of world – line supergravity, $\mathcal{A}$ is the background Maxwell superfield and l is the nonlocality parameter having the dimension of length. Such form of the action is very convenient for the application to the case of background supergravity considered in the next section.

The action (10) is closely related to one proposed in ref. [20]. In the paper of Delduc and Sokatchev the conventional background constaints (namely (15), (16)) playing the role of the integrability conditions and maintaining original theory symmetries in the presence of background interaction was obtained in elegant manner. The presence of the additional term does not change this situation that gives a possibility to consider usual constraints imposed to the Maxwell and supergravity background superfield.

Twistor shift is generated by transition to the new space – time variables

$$\hat{X}^{\alpha\beta} = X^{\alpha\beta} + \frac{l}{2\tilde{\mu} D\Theta} (\tilde{\mu}^\alpha D\Theta^\beta + \tilde{\mu}^\beta D\Theta^\alpha) \qquad (12)$$

where $\tilde{\mu}^\alpha = \mu^\alpha + \eta d^\alpha$ is the even superfield containing the second twistor half μ and its superpartner d, moreover,

$$\tilde{\mu}_\alpha = iX_{\alpha\beta}D\Theta^\beta$$

By using the equation of motion in the first order over l after the twistor shift we obtain the following action modulo term $D\Theta^\alpha\mathcal{A}_\alpha$ which we shall consider below

$$\tilde{S} = \int d\tau\, d\eta\, (iE^{-1}D\Theta\gamma_m D\Theta D\hat{X}^m + \hat{\Omega}^m\mathcal{A}_m + \frac{1}{2}l\hat{\Omega}^{\alpha\beta}(\sigma^{mn})_{\alpha\beta}\mathcal{F}_{mn}) \tag{13}$$

where $\hat{\Omega}^m$ is the Cartan form invariant under local "small" and global "big" SUSY transformations

$$\hat{\Omega}^m = D\hat{X}^m + i\Theta\gamma^m D\Theta + iD\Theta\gamma^m\Theta$$

.

In D=3 the vector field is a part of spinor supermultiplet

$$\mathcal{A}_\alpha = r_\alpha + B\Theta_\alpha + V_{\alpha\beta}\Theta^\beta + h_\alpha\Theta\Theta \tag{14}$$

where $V_{\alpha\beta} = V^m(\gamma_m)_{\alpha\beta}$ and h is the vector gauge field and its superpartner, respectively.

The Wess – Zumino gauge $r = B = 0$ and conventional constraints

$$\mathcal{F}_{\alpha\beta}(\hat{X}, \Theta) = 0 \tag{15}$$

$$T^a_{\alpha\beta}(\hat{X}, \Theta) = 2i\gamma^a{}_{\alpha\beta} \tag{16}$$

extract irreducible submultiplet of physical fields, after that the action (13) may be present in the form of

$$\tilde{S} = \int d\tau\, d\eta\, iE^{-1}D\Theta\gamma_m D\Theta D\hat{X}^m - \frac{i}{2}\hat{\Omega}^m(V_m + \frac{1}{2}\Theta_\beta(\gamma_m)^{\beta\alpha}h_\alpha - \frac{i}{2}\Theta\Theta\varepsilon^{kl}{}_m F_{kl})$$
$$+ i\frac{1}{2}l\varepsilon^{nmk}\hat{\Omega}_k(F_{nm} + \Theta_\alpha(\gamma_n)^{\alpha\beta}\partial_m h_\beta - \frac{i}{2}\Theta\Theta\varepsilon^{pl}{}_m\partial_n F_{pl}) \tag{17}$$

where ε_{mnk} is the Levi – Chivita tensor;F^{mn} is the Maxwell field strength tensor.

Integrating over η and fixing solution of the equation of motion

$$\frac{\delta L}{\delta \hat{\psi}} = 0$$

in the form of

$$\chi^m = -(\theta\gamma^m\lambda + \lambda\gamma^m\theta) \tag{18}$$

we come to the component form of the Lagrangian (17). Such choosing of the solution connected with the first term of the action (17) has been considered by Volkov and Zheltukhin [3].

It is easy to see writing down the remaining term of the action (10) into the component form and using the equation of motion

$$\frac{\delta L}{\delta e} = 0$$

that this one duplicates some terms of the action (17) so that full Lagrangian after the twistor shift appears as

$$\tilde{L} = -(e^{-1}\lambda\gamma_m\lambda - (V_m + \frac{1}{2}\theta_\beta\gamma_m^{\beta\alpha}h_\alpha - \frac{i}{4}\theta\theta\varepsilon^{kl}{}_m F_{kl})$$
$$+ l\varepsilon^{nk}{}_m(F_{nk} + \frac{1}{2}\theta_\alpha\gamma_n^{\alpha\beta}\partial_k h_\beta - \frac{i}{4}\theta\theta\varepsilon^{pl}{}_k\partial_n F_{pl}))\dot{\omega}^m \tag{19}$$

where ω^m is the Cartan differential form

$$d\omega^m = d\hat{x}^m - i\theta\gamma^m d\theta + id\theta\gamma^m\theta + 2\lambda\gamma^m\lambda d\tau \tag{20}$$

Lagrangian (19) is invariant under the following transformations of the global SUSY on the mass – shell

$$\delta V_m = -\varepsilon_\beta(\gamma_m)^{\beta\alpha}h_\alpha;\ \delta\Theta_\beta = \varepsilon_\beta;\ \delta h_\alpha = \frac{i}{2}\varepsilon^\beta(\sigma^{nm})_{\beta\alpha}F_{nm}$$

with odd parameter ε.

Now let us consider the algebra of the constraints obtaining from the action (19). In the case of usual Brink – Schwarz superparticle this one has the following form:

$$\{d_\alpha, d_\beta\} = -2(\gamma^a)_{\alpha\beta}p_a$$

where d_α and d_β are the second class constraints and p_a is the covariant superparticle momentum transforming under the quantization procedure into the covariant vector derivative. In our case

$$d_\alpha = P_{\theta^\alpha} - (e^{-1}\lambda\gamma_m\lambda - \tilde{V}_m)\gamma^m\theta_\alpha$$

where

$$\tilde{V}_m = V_m + \frac{1}{2}\theta_\beta\gamma_m^{\beta\alpha}h_\alpha - \frac{i}{4}\theta\theta\varepsilon^{kl}{}_m F_{kl}$$
$$+l\varepsilon^{nk}{}_m(F_{nk} + \frac{1}{2}\theta_\alpha\gamma_n^{\alpha\beta}\partial_k h_\beta - \frac{i}{4}\theta\theta\varepsilon^{pl}{}_k\partial_n F_{pl})$$

and it is easy to see that the algebra of constraints does not change upon applying the twistor shift procedure in the first order in l expansion with vector covariant derivative having modification like (9), moreover, we hope that it is remain true for any order of l^n expansion. The main reason of preserving the algebraic structure is the conventional constraint (15) reproducing the integrability condition.

Thus, the twistor shift procedure allows to get rid of undesirable term $\dot{\theta}\dot{\theta}$ breaking the spin – statistic connection under turning into the field theory. This leads to arising in bosonic sector an infinite series in the parameter l containing Maxwell strength tensor and its higher derivatives. The fermionic sector present in the case of superparticle is also modified by terms containing derivatives from a gauge field superpartner. The algebra of second class constraints does not change upon the twistor shift procedure due to imposing conventional constraint playing the role of integrability condition.

4 Supertwistor shift in the presence of the background supergravity.

The more complicated case is, of course, the case of the supergravity background. We would like firstly to discuss this one in general and then to apply this consideration to our approach.

The spinorial covariant derivatives of the D=3 N=1 supergravity in the Wess – Zumino gauge have the following form [16, 17, 18]:

$$D_\alpha = \partial_\alpha - i\frac{1}{2}(\gamma^0\gamma^a\Theta)_\alpha \nabla_a + \frac{1}{4}\Theta\Theta(\gamma^0\hat{\nabla})_\alpha \tag{21}$$

where

$$\begin{gathered}\nabla_a = \psi_a^\mu \partial_\mu + e_a^m \partial_m + \frac{1}{2}\omega_a^{kp} M_{kp} \\ \hat{\nabla}^\alpha = \nabla^\alpha + \frac{1}{4}\omega_a^{kp}\varepsilon_{kp}{}^d(\gamma^a\gamma_d\partial)^\alpha + \frac{1}{2}(\gamma^a\gamma^b\psi_a)^\alpha \nabla_b \\ \nabla^\alpha = (\phi\gamma^0)^{\alpha\mu}\partial_\mu + \chi^{\alpha a}\nabla_a + \frac{1}{2}\zeta^{\alpha kp}M_{kp}\end{gathered} \tag{22}$$

In our notations $\partial_m = \partial/\partial X^m$, $\partial_\alpha = \partial/\partial\Theta^\alpha$, $(\gamma^0)_{\alpha\beta} = \varepsilon_{\alpha\beta}$ in Majorana representation of the γ – matrices (see Appendix), $\varepsilon_{kp}{}^d$ is the Levi – Chivita tensor; e_a^m, ω_a^{kp}, ψ_a^μ are the components of the dreinbein, connection and gravitino respectively; the structure group of the tangent space is the $SL(2,R)$ with the Lorentz generators M_{kp}. The remaining fields provide closing the SUSY algebra on the "off – shell".

With the help of the expression (21) and (22) we can restore the superdreinbein components by using of

$$D_A = E_A^M \partial_M + W_A{}^{kp} M_{kp} \tag{23}$$

where indices $A = \{a, \alpha\}$ take a value of the vector and spinor indices in the tangent "flat" superspace, $M = \{\underline{m}, \mu\}$ are analogous indices of the "curve" superspace, $W_A{}^{kp}$ is the superconnection taking value in the structure group.

As in the case of the Maxwell background it is necessary to impose the conventional constraints on the curvature and torsion consistent with the Wess – Zumino gauge [18]:

$$\begin{gathered}\mathcal{R}_{\alpha\beta}(X,\Theta) = 0 \\ T^a_{\alpha\beta}(X,\Theta) = -\frac{i}{2}\gamma^a{}_{\alpha\beta}\end{gathered}$$

so that

$$\{D_\alpha, D_\beta\} = -i\gamma^a_{\alpha\beta} D_a \tag{24}$$

Then from (21) and (22) we obtain

$$
\begin{aligned}
E^{\mu}_{\alpha} &= \delta^{\mu}_{\alpha} - i\frac{1}{2}(\gamma^0\gamma^a\Theta)_{\alpha}\psi^{\mu}_{a} + \frac{1}{4}\Theta\Theta(\gamma^0)_{\alpha\beta}\{(\phi\gamma^0)^{\beta\mu} \\
&\quad + \frac{1}{2}(\gamma^a\gamma^b\psi_a)^{\beta}\psi^{\mu}_{b} + \frac{1}{4}\omega^{kp}_{a}\varepsilon_{kp}{}^{d}(\gamma^a\gamma_d)^{\beta\mu}\} \qquad (25)\\
E^{\underline{m}}_{\alpha} &= -i\frac{1}{2}(\gamma^0\gamma^a\Theta)_{\alpha}e^{\underline{m}}_{a} + \frac{1}{8}\Theta\Theta(\gamma^0)_{\alpha\beta}(\gamma^a\gamma^b\psi_a)^{\beta}e^{\underline{m}}_{b}
\end{aligned}
$$

and from (24) and

$$D_a = E_a{}^{\underline{m}}\partial_{\underline{m}} + E_a{}^{\mu}\partial_{\mu} + W_a{}^{kp}M_{kp}$$

we get the expression for the remaining superdreinbein components:

$$
\begin{aligned}
E^{\underline{m}}_{a} &= e^{\underline{m}}_{a} + \frac{1}{4}i\Theta\gamma_a\psi^{\underline{m}} - \frac{1}{8}\Theta\Theta(\gamma^b)_{\alpha\gamma}\psi^{\gamma}_{b}(\gamma^c)^{\alpha\varepsilon}\psi_{a\varepsilon}e^{\underline{m}}_{c} \\
&\quad + \frac{1}{4}\Theta\Theta\phi e^{\underline{m}}_{a} - \frac{1}{8}\Theta\Theta\omega^{kp}_{a}Tr(\sigma_{kp}\gamma^c)e^{\underline{m}}_{c} \\
E^{\mu}_{\underline{n}} &= \psi^{\mu}_{\underline{n}} + \frac{1}{4}i\Theta_{\alpha}(\gamma^a)^{\alpha\beta}\psi_{\underline{n}\beta}\psi^{\mu}_{a} - \frac{1}{4}i\Theta_{\alpha}(\sigma_{kp})^{\alpha\mu}\omega^{kp}_{\underline{n}} \\
&\quad + \frac{1}{8}\Theta\Theta(\gamma^a)_{\alpha\gamma}\psi^{\gamma}_{a}\omega^{kp}_{\underline{n}}(\sigma_{kp})^{\alpha\mu} + \frac{1}{4}\Theta\Theta\phi\psi^{\mu}_{\underline{n}} - \frac{1}{8}\Theta\Theta\omega^{kp}_{\underline{n}}Tr(\sigma_{kp}\gamma^a)\psi^{\mu}_{a} \qquad (26)
\end{aligned}
$$

The last terms of these exrpessions vanish by virtue of $Tr(\sigma_{kp}\gamma^c) = 0$, where Tr denotes the trace and σ_{kp} is the antisymmetric product of Dirac matrices (see Appendix).

Since the twistor shift procedure is carried out on the "mass – shell" we can neglect the auxiliary scalar field contribution.

The action describing superparticle in the gravitational background field has the following form [20]:

$$S = -i\int d\tau\, d\eta\, E^{-1}DE^{\alpha}DE^{\beta}(\gamma_m)_{\alpha\beta}DE^{m} \qquad (27)$$

Using $DE^A = (Dz^M)E^A_M$ we obtain the following expression for the Lagrangian:

$$
\begin{aligned}
L &= D\Theta^{\lambda}E^{\alpha}_{\lambda}D\Theta^{\gamma}E^{\beta}_{\gamma}DX^{\underline{k}}E^{m}_{\underline{k}}(\gamma_m)_{\alpha\beta} + 2D\Theta^{\lambda}E^{\alpha}_{\lambda}DX^{\underline{n}}E^{\beta}_{\underline{n}}D\Theta^{\gamma}E^{m}_{\gamma}(\gamma_m)_{\alpha\beta} \\
&\quad + (\gamma_m)_{\alpha\beta}(2D\Theta^{\lambda}E^{\alpha}_{\lambda}DX^{\underline{n}}E^{\beta}_{\underline{n}}DX^{\underline{k}}E^{m}_{\underline{k}} + DX^{\underline{m}}E^{\alpha}_{\underline{m}}DX^{\underline{n}}E^{\beta}_{\underline{n}}D\Theta^{\lambda}E^{m}_{\lambda}) \qquad (28)
\end{aligned}
$$

(By using the superreparametrization we always can choose $E^{-1} = 1$).

It is easy to prove that the last two terms of the (28) vanish (for this aim it is nessesary to write down these terms in the component form) and after that, using (25) and (26) we can find the remaining Lagrangian

$$L = D\Theta^{\lambda} D\Theta^{\gamma} DX^{\underline{n}} (\gamma_m)_{\alpha\beta} \{ \delta^{\alpha}_{\lambda} \delta^{\beta}_{\gamma} e^{m}_{\underline{n}} + \frac{1}{4} i \delta^{\alpha}_{\lambda} \delta^{\beta}_{\gamma} \Theta \gamma^m \psi_{\underline{n}} - i(\gamma^k \Theta)_{\gamma} \psi^{\beta}_{k} \delta^{\alpha}_{\lambda} e^{m}_{\underline{n}}$$
$$+ \frac{1}{2} \Theta\Theta \delta^{\alpha}_{\lambda} (\frac{1}{2} (\gamma^a \gamma^b \psi_a)_{\gamma} \psi^{\beta}_{b} e^{m}_{\underline{n}}) - \frac{1}{8} \delta^{\alpha}_{\lambda} \delta^{\beta}_{\gamma} \Theta\Theta (\gamma^b)_{\alpha\gamma} \psi^{\gamma}_{b} (\gamma_c)^{\alpha\varepsilon} \psi^{m}_{\varepsilon} e^{c}_{\underline{n}} \}$$
$$+ 2 D\Theta^{\lambda} D\Theta^{\gamma} DX^{\underline{n}} (\gamma_m)_{\alpha\beta} \{ -\frac{i}{2} \delta^{\alpha}_{\lambda} \psi^{\beta}_{\underline{n}} (\gamma^m \Theta)_{\gamma} + \frac{1}{8} \delta^{\alpha}_{\lambda} (\Theta \gamma^c \psi_{\underline{n}}) \psi^{\beta}_{c} (\gamma^m \Theta)_{\gamma} \tag{29}$$
$$- \frac{1}{4} (\gamma^k \Theta)_{\lambda} \psi^{\alpha}_{k} \psi^{\beta}_{\underline{n}} (\gamma^m \Theta)_{\gamma} + \frac{1}{8} \Theta\Theta \delta^{\alpha}_{\lambda} \psi^{\beta}_{\underline{n}} (\gamma^a \gamma^m \psi_a)_{\gamma} \}$$

With help of the evident identity

$$\Theta^{\alpha} \Theta^{\beta} = -\frac{1}{2} \varepsilon^{\alpha\beta} \Theta\Theta$$

the Lagrangian (29) is reduced to the following form

$$L = D\Theta^{\lambda} D\Theta^{\gamma} DX^{\underline{n}} (\gamma_m)_{\alpha\beta} \{ \delta^{\alpha}_{\lambda} \delta^{\beta}_{\gamma} e^{m}_{\underline{n}} + \frac{1}{4} i \delta^{\alpha}_{\lambda} \delta^{\beta}_{\gamma} \Theta \gamma^m \psi_{\underline{n}} \tag{30}$$
$$+ \frac{1}{4} \Theta\Theta \delta^{\alpha}_{\lambda} (\gamma^a \gamma^m \psi_a)_{\gamma} \psi^{\beta}_{\underline{n}} \}$$

and describes the superparticle interacting with the background D=3 N=1 supergravity.

The generalization of the dynamics is achieved by introduction of the additional term

$$L_{additional} = -il \dot{E}^{\alpha} D E_{\alpha} \tag{31}$$

having an extended expression in the form of

$$L_{additional} = -il \{ (\dot{z}^N D z_M (-)^{(\alpha+N)(M+1)} E^{\alpha}_{N} E^{M}_{\alpha} \tag{32}$$
$$+ (-)^{\alpha(N+1)} (D z^N)(D z_M) E^{M}_{\alpha} (D E^{\alpha}_{N}) \}$$

Since the expression (32) contains z^N we can consider the contribution of X^m variables, but for investigation of our system it is neseccary to choose namely $z^{\mu} \equiv \Theta^{\mu}$ (accounting X^m leads to considering the spinning superparticle [13] interacting with the background supergravity). Then

$$L_{additional} = -il (\dot{\Theta}^{\nu} D\Theta_{\mu} E^{\alpha}_{\nu} E^{\mu}_{\alpha} + D\Theta^{\nu} D\Theta_{\mu} DX^{\underline{k}} (\tilde{\Omega}_{\underline{k}})^{\mu}_{\nu}) \tag{33}$$

where $(\tilde{\Omega}_{\underline{k}})^{\mu}_{\nu}$ is the object of unholonomity having only the following non-vanishing components

$$(\tilde{\Omega}_{\underline{k}})^{\mu}_{\nu} = (\tilde{\Omega}^{a}_{bc}) e_{a\underline{k}} (-\frac{1}{2}) (\sigma^{bc})^{\mu}_{\nu} \tag{34}$$

Imposing the additional torsion constraint

$$T^{a}_{bc} = 0 \tag{35}$$

makes it possible to express the superconnection Ω through the $\tilde{\Omega}$ as:

$$\tilde{\Omega}^{a}_{bc} = -2\Omega^{a}{}_{[bc]} \tag{36}$$

where the braces $[\ldots]$ denote an antisymmetrization.

The connection of the unholonomity object with the superdreinbein components is well known:

$$\tilde{\Omega}^{A}_{BC} = (-)^{B(M+C)} V^{M}_{C} V^{N}_{B} (E^{A}_{M,N} - (-)^{MN} E^{A}_{N,M}) \tag{37}$$

where $E^{A}_{M,N} = \partial E^{A}_{M} / \partial z^{N}$.

Then

$$\begin{aligned}
\tilde{\Omega}^{a}_{bc} = {} & e^{\underline{m}}_{c} e^{\underline{n}}_{b} (\partial_{\underline{n}} e^{a}_{\underline{m}} + \frac{1}{4} i\theta\gamma^{a} \partial_{\underline{n}} \psi_{\underline{m}} - (\underline{m} \longleftrightarrow \underline{n})) \\
& + \frac{1}{4} i\theta\gamma_{b} \psi^{\underline{n}} e^{\underline{m}}_{c} (\partial_{\underline{n}} e^{a}_{\underline{m}} - (\underline{m} \longleftrightarrow \underline{n})) + \frac{1}{4} i\theta\gamma_{b} \psi^{\underline{n}} e^{\underline{m}}_{c} (\frac{1}{4} i\theta\gamma_{a}) (\partial_{\underline{n}} \psi_{\underline{m}} - (\underline{m} \longleftrightarrow \underline{n})) \\
& + \frac{1}{4} i\theta\gamma_{c} \psi^{\underline{m}} e^{\underline{n}}_{b} (\partial_{\underline{n}} e^{a}_{\underline{m}} - (\underline{m} \longleftrightarrow \underline{n}))) + \frac{1}{4} i\theta\gamma_{c} \psi^{\underline{m}} e^{\underline{n}}_{b} (\frac{1}{4} i\theta\gamma_{a}) (\partial_{\underline{n}} \psi_{\underline{m}} - (\underline{m} \longleftrightarrow \underline{n})) \\
& - \frac{1}{16} (\theta\gamma_{c} \psi^{\underline{m}}) (\theta\gamma^{b} \psi^{\underline{n}}) (\partial_{\underline{n}} e^{a}_{\underline{m}} - (\underline{m} \longleftrightarrow \underline{n})) + \frac{1}{4} \theta\theta e^{\underline{m}}_{c} e^{\underline{n}}_{b} (\frac{1}{2} (\gamma^{b} \gamma^{a} \psi_{b})_{\beta} (\partial_{\underline{n}} \psi^{\beta}_{\underline{m}} - (\underline{m} \longleftrightarrow \underline{n}))) \\
& - \frac{1}{8} \theta\theta \{ e^{\underline{m}}_{c} (\gamma^{a} \gamma_{b} \psi_{a})^{\beta} \psi^{\underline{n}}_{\beta} + e^{\underline{n}}_{b} (\gamma^{a} \gamma_{c} \psi_{a})^{\beta} \psi^{\underline{m}}_{\beta} \} (\partial_{\underline{n}} e^{a}_{\underline{m}} - (\underline{m} \longleftrightarrow \underline{n}))
\end{aligned} \tag{38}$$

Now let us turn back to the (30), (33) and carry out the twistor shift by replacing of the space – time coordinates

$$\hat{X}_{\underline{m}} = X_{\underline{m}} + \frac{l}{2D\Theta\tilde{\mu}} D\Theta^{\alpha} (\gamma_{a})_{\alpha}{}^{\beta} \tilde{\mu}_{\beta} E^{a}_{\underline{m}} \tag{39}$$

that leads to arising a new term

$$-i\frac{1}{2} l \varepsilon_{\underline{k}}{}^{\underline{nl}} \partial_{\underline{n}} E^{m}_{\underline{l}} (\gamma_{m})_{\alpha\beta} D\Theta^{\alpha} D\Theta^{\beta} DX^{\underline{k}}$$

Then after cumbersome calculations using Volkov – Zheltukhin solution [3] we obtain the Lagrangian describing a generalized twistor dynamics of superparticle interacting with the D=3 N=1 supergravity background field

$$
\begin{aligned}
L^{shift}_{SG} &= \lambda^\alpha\lambda^\beta(\gamma_a)_{\alpha\beta}\{e^a_{\underline{m}} + \frac{1}{4}i\theta\gamma^a\psi_{\underline{m}} + \frac{1}{4}\theta\theta(\gamma^b\gamma^a\psi_b)_\gamma\psi^\gamma_{\underline{m}} \\
&+ \frac{1}{4}l\varepsilon_{\underline{m}}{}^{\underline{nl}}(T^a_{\underline{nl}} + \frac{1}{4}i(\theta\gamma^a)_\rho T^\rho_{\underline{nl}} + \frac{1}{4}\theta\theta(\gamma^b\gamma^a\psi_b)_\rho T^\rho_{\underline{nl}})\}\dot{\omega}^{\underline{m}} \qquad (40)
\end{aligned}
$$

Here all fields depend on the new space – time variables $\hat{x}$, $T^a_{\underline{nl}}$ and $T^\rho_{\underline{nl}}$ are the torsions connected with the dreinbein and gravitino respectively, $\omega^{\underline{m}}$ is the superCartan form. In the limiting case of the $\theta = 0$ our Lagrangian reproduces the result of ref. [7].

The investigation of the constraints algebra is analogous to the Maxwell field interaction case. Again due to the existence of the conventional constraint

$$\mathcal{R}_{\alpha\beta}(\hat{X}, \Theta) = 0$$

the algebra of the second class constraints does not change.

Thus, in the framework of generalized twistor dynamics, superparticle interacting with background supergravity "feels" space torsion and, in the second order over l, curvature.

In the end of this section we would like to make some remarks concerning superparticle interacting with background fields. The demanding preservation of all original symmetries imposes some conditions on background fields. In particular, the preservation of the Siegel fermionic symmetry in the presence of a gauge field requires that the superfield strength satisfies the super – Yang – Mills equations of motion (see ref. [23] and refs. therein). It is achieved by imposing conventional constraints (in the case of Maxwell background field these ones are (15) and (16)).

For the superobject in a general curved superspace, the situation is more complicated. In the standard formulation of N=1 Brink – Schwarz superparticle the action describing the former looks like

$$S = \int d\tau \frac{1}{2}e^{-1}\eta_{ab}\dot{z}^M E^a_M \dot{z}^N E^b_N$$

and it is easy to note that superparticle does not "feel" E^{α} components of background supervierbein. For this reason the preserving original symmetries does not lead to the superfield supergravity equation of motion but to the classes of equivalence of supergravity background fields.

In contrast to the standard version twistor – like superparticle "feels" all components of background supervierbein and it would be very interesting to investigate conditions imposed on background supergravity first steps to the studying of which have been made in refs. [20, 12].

5 Conclusion.

In the first time it was assumed to use the twistor shift procedure for the solution of the bosonic string tachion sector problem. This sector arises on the particle level when we try to carry out a generalization of twistor dynamics based on the STVZ action. Unfortunately, there are not any successes on this direction as well as for attempts of consideration twistor shift in the case of higher dimensions D=4,6. Thus, the main application of the twistor shift connected with anyon physics in D=3.

The existence of twistor shift procedure touches upon very important question concerning the meaning of physical coordinates in general and connection between worldline (worldsheet) locality and causality with target space ones. With consideration of generalized twistor dynamics we expand phase space by inclusion of additional variables λ and their momenta. Upon the twistor shift procedure we eliminate additional phase space coordinates, but it does not mean that we return explicitly to the original phase space. The absence of clear understanding of this situation does not allow us to make hard conclusion about, for example, structure of constraints algebra, which closely connected with conventional constraints imposed on background fields. We naively assume that this constraints have an anologous form as in the standard case, but it may be not really so !

From the other point of view the expanded phase space of a generalized twistor dynamics possesses a noncomutativity of the space – time coordinates, restored by twistor shift. However in the last time there are many consideration of the theories with noncommutative coordinates on the Plank scale. Of course, the theories of such kind have many difficulties connected, for example, with the ordering of the coordinates, introduction of interaction with background fields etc., but it would be very interesting to investigate a generalized twistor dynamics without twistor shift in this context.

Thus, a generalized twistor dynamics puts principal and deep questions and stimulates the finding of their solution.

Acknolegments.

The authors are grateful to D.V.Volkov (A.N. personally thank D.V. for the fruitfull discussions of these problems and great help in preparation of this article to publication) and also D.P.Sorokin and I.A.Bandos for stimulating discussions. A.N. acknowledge support from International Soros Foundation, Grant N RY 9000 and partial support from Ukranian State Committee in Science and Technology, Grant N 2.3/664. V.T. thanks to METU (Ankara) for the kind hospitality and T.Aliev for useful discussions. V.T. acknowledge support from International Soros Foundation, Grant N U9G000.

Appendix.

We choose a real (Majorana) representation for the Dirac matrices $(\gamma_m)_\alpha^\beta$ and charge conjugation matrix $C_{\alpha\beta}$ (D=3):

$$\gamma^0 = C = -i\sigma_2, \gamma^1 = \sigma_1, \gamma^2 = \sigma_3, C_{\alpha\beta} = \varepsilon_{\alpha\beta}$$

where σ_i are the Pauli matrices. γ – matrices satisfy the following relations:

$$\{\gamma_m, \gamma_n\} = 2g_{mn}, g_{mn} = diag(-,+,+)$$

$$\sigma_{mn} = \frac{1}{4}[\gamma_m, \gamma_n] = -\frac{1}{2}\varepsilon_{mnl}\gamma^l$$
$$2\delta_\alpha^\beta \delta_\gamma^\delta = \delta_\alpha^\delta \delta_\gamma^\beta + (\gamma^m)_\alpha^\delta (\gamma_m)_\gamma^\beta$$

where ε^{mnl} is the Levi – Chivita tensor and $(\varepsilon^{012} = 1)$. The vector and spin – tensor representations connect as

$$x_\alpha^\beta = (\gamma^m x_m)_\alpha^\beta$$

Raising and lowering of spinor indices is carried out by the matrix $\varepsilon_{\alpha\beta}$ according to the rules:

$$\lambda_\alpha = \varepsilon_{\alpha\beta}\lambda^\beta, \lambda^\alpha = \varepsilon^{\alpha\beta}\lambda_\beta, \varepsilon_{12} = -\varepsilon^{12} = -1.$$

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